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Large mixing, family structure, and dominant block in the neutrino mass matrix

Francesco Vissani

Deutsches Elektronen-Synchroton, DESY
Notkestraße 85, D-22603, Hamburg, Germany
E-mail: vissani@mail.desy.de

Abstract: A possible connection between the flavour structure of the charged fermions and the large $\nu_\mu - \nu_\tau$ mixing motivates an ansatz for the neutrino mass matrix with a dominant block. We distinguish between a general form and the specific forms of the ansatz, and concentrate on the cases of phenomenological interest. The general form can incorporate an observable amount of CP violation in the leptonic sector. Only specific forms can incorporate the Mikheyev-Smirnov-Wolfenstein solutions for solar neutrinos, with small or large mixing angles, or, alternatively, vacuum oscillation solutions. Other specific variants explain the Los Alamos neutrino anomaly, or provide a two-neutrino hot dark matter component.

Keywords: Neutrino Physics, Solar and Atmospheric Neutrinos, CP violation, Dark Matter.

Contents

1. General form of the ansatz

1.1. Definition and theoretical motivation

Recently, a simple ansatz for neutrino masses was proposed [1, 2]:

$$\mathcal{L}_{\nu\ mass} \sim -m_\nu\ (\nu_\tau, \nu_\mu, \nu_e) \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \nu_\mu \\ \nu_e \end{pmatrix} + h.c.\,; \tag{1.1}$$

$\nu_{\tau,\mu,e}$ are the neutrinos of a given flavour, ϵ is a small parameter discussed below. The neutrino mass scale m_ν can be expressed as:

$$m_\nu = \frac{v^2}{M_{heavy}}\,, \qquad v = 174\ \text{GeV}\,,$$

where M_{heavy} is a heavy mass scale, suggestive of a seesaw explanation of the smallness of the neutrino masses [3]. The elements of the mass matrix are specified up to (generally complex) coefficients of order of unity; whenever these coefficients affect a relation, we will use the symbols $\sim$, $\gtrsim$ or $\lesssim$. The ansatz is characterized by a "dominant block" in the ν_μ, ν_τ subspace, in the sense that the elements of this block are much larger than the others, as accounted by the parameter ϵ, and are comparable among them.

The underlying idea of this structure for the mass matrix is to identify ν_μ and ν_τ, but to distinguish them from ν_e, in a model where known characteristics of the charged fermions are also reproduced. It was suggested [1] that in SU(5) context, two matter fields $\bar{5} = (L, D^c)$ may have the same flavour properties (nonparallel family structure, in the following "NFS model"). A single parameter ϵ describes the flavour structure of charged fermions:

$$\epsilon = 1/20 \simeq m_\mu/m_\tau\,. \qquad \text{(NFS value)}\,. \tag{1.2}$$

The same goal was achieved by an anomalous U(1) family symmetry under which ν_μ and ν_τ have the same charge [2] (denoted "ABE model", from the abelian symmetry), which suggested instead the identification with a power of the Cabibbo angle:

$$\epsilon = 8 \cdot 10^{-3} \simeq (\sin\theta_C)^3 \qquad \text{(ABE value)}\,. \tag{1.3}$$

The ansatz is assigned in the flavour basis, where the charged leptons are mass eigenstates, and the leptonic charged currents are diagonal: In fact, this basis is the relevant one for a discussion of the phenomenological implications. Let us however consider closely the theoretical motivations. The postulated identity of the muon and tau neutrino properties together with SU(2) gauge invariance imply that also μ and τ have identical properties, in the basis where the ansatz is originally assigned. In particular, $\tau\tau^c$ and $\mu\tau^c$ should get comparable mass entries, and therefore the original basis is not the flavour basis: A rotation on the "left" charged leptons with large angle (more precisely, a redefinition) is necessary to eliminate the term $\mu\tau^c$. This happens in the specific models [1, 2]; see also [4]. The *assumption* that this rotation does not spoil (or seriously affect) the ansatz imply a restriction on the coefficients of order unity. These considerations illustrate the importance of a detailed theory of the coefficients of order unity (that at present is lacking) already at the level of theoretical justification of the ansatz.

The simplicity of the neutrino mass matrix (1.1) is however remarkable and perhaps of profound meaning. We will discuss this ansatz and its implications in the two models defined above, and in more generic cases (that is, treating m_ν and ϵ as parameters—the latter is always assumed to be small in comparison with unity). Our aim in particular is: (a) to disentangle the generic and specific features of the ansatz (1.1), and (b) to identify its predictions.

1.2. Features

If the coefficients order unity should be all equal, the mass matrix in eq. (1.1) would have only one massive eigenstate, mostly a combination of the muon and tau neutrino flavour eigenstates, whereas the other two states would be massless. This is modified in the general case. There are three typical features:

i) There are two massive neutrinos, ν_2 and ν_3, with masses of order m_ν (=the *no-singularity* hypothesis) and with a comparable mass splitting (=the *no-degeneracy* hypothesis); the lighter state ν_1 has mass order $m_\nu \cdot \epsilon^2$.

ii) The muon and tau neutrinos are mostly combinations of the two heavy states, with a large mixing angle $\theta \simeq 45^o$.

iii) The electron neutrino is almost coincident with the lightest state ν_1, and has mixing angles of order $\theta \sim \epsilon$ with the heavier states (in another notation, $|U_{e3}| \sim |U_{e2}| \sim \epsilon$):

$$\sin^2(2\theta) \sim (2\epsilon)^2 \simeq 10^{-2} \text{ or } 2.5 \cdot 10^{-4}, \text{ for NFS or ABE model}. \qquad (1.4)$$

Feature *i*) suggests that the splitting between the two heavier neutrinos is $\sim m_\nu^2$ unless the *no-degeneracy* hypothesis is not fulfilled. Therefore, assuming that $M_{heavy} \sim 7{\cdot}10^{14}$ GeV, it is possible to incorporate the mass splitting necessary for the explanation of atmospheric neutrinos: $\Delta m^2_{atm} \simeq 2 \cdot 10^{-3}$ eV^2. Unfortunately, it is not possible to be more specific and *predict* whether $\Delta m^2_{atm} > 2 \cdot 10^{-3}$ eV^2 or the contrary is true (which is of crucial importance for the success of the K2K experiment [5, 6]). The large mixing angle instead comes automatically due to *ii*), and this is the most attractive feature of the ansatz. In fact, this permits to explain the atmospheric neutrino anomaly [7] in terms of neutrino oscillations [8]. The mixing (1.4) is quite close to the small mixing angle necessary for the adiabatic Mikheyev-Smirnov-Wolfenstein (MSW) solution [9] of the solar neutrino problem, or to the angle required to explain the LSND anomaly [10] (with NFS value). But, generically, also the mass splittings with the lighter neutrino are also $\sim m_\nu^2$, for the *no-singularity* hypothesis: therefore, accounting for oscillations of atmospheric neutrinos, the solar or LSND neutrino anomalies remain unexplained. We will discuss in sections 2 and 3 the specific cases in which this conclusion does not hold.

1.3. Leptonic CP violation

However, one could take a radically theoretical point of view, and suggest (based on the ansatz, and on the mass scale chosen) that only the atmospheric neutrinos *should* be explained by neutrino oscillations. This option would probably become more attractive if the results of the chlorine experiments could not be reproduced, the SNO experiment [11] failed to detect an excess of neutral current induced events,[1] the theoretical estimations for solar neutrinos were much more uncertain than expected, and the LSND anomaly was not confirmed by other experiments. The appealing feature of having two large mass splittings is the possibility to search for CP-violation in the leptonic sector in terrestrial experiments [13, 14], for instance by comparing the probabilities of conversion:

$$P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e) = 4J_{lept}\ \sin\varphi_{21} \sin\varphi_{31} \sin\varphi_{32}\,.$$

[1] An alternative interpretation would be in terms of existence of sterile neutrinos [12].

The phases of oscillations in vacuum $\varphi_{ij} = \Delta m^2_{ij} L/(4E_\nu)$ are comparable among them, and of interest for terrestrial experiments since: $\Delta m^2_{ij} \sim \Delta m^2_{atm}$. The leptonic analogue of the Jarlskog invariant,

$$J_{lept} = \mathrm{Im}(U^*_{\mu 1} U_{e1} U_{\mu 2} U^*_{e2})\,,$$

is suppressed due to the presence of two small mixings (eq. (1.4)): $J_{lept} = \mathcal{O}(\epsilon^2)$ (or smaller, depending on the phases). Observable effects in forthcoming experiments would imply:

$$\epsilon \gtrsim 0.1\,,$$

that is not far from the value in the NFS model (1.2), but much larger than the value (1.3).

Another feature of the models (1.2) and (1.3), is that the electron neutrinos are not expected to undergo any significant oscillation in the atmosphere. Hence, the search for a subdominant mixing by using the lowest energy neutrino-induced events [15, 16, 17, 18] will also test the mass matrix (1.1). The present bound $|U_{e3}|^2 < 0.15$ at 90 % CL, obtained in a model with a single neutrino ν_3 split in mass [15], suggests the limit:

$$\epsilon^2 \lesssim 0.1\,.$$

This is far also from (1.2). (Notice that we *estimated* the sensitivity of the atmospheric neutrino studies to small mixings; but, in order to assess precise bounds, it is necessary to go beyond the usually [15, 16, 17, 18] kept hypothesis of degenerate ν_1 and ν_2.)

We offer a theoretical guess for a large value of ϵ, compatible with both previous bounds, and suggestive of a link between the neutrino mass matrix and the flavour structure:

$$\epsilon = 0.2 \simeq \sin\theta_C \qquad (\text{“large” value})\,. \tag{1.5}$$

A component of oscillation $|U_{e3}|$ of this size could be detectable, studying the dependence on the energy cuts of the observed event rates at the Super-Kamiokande (SK) [17].

2. Specific forms of the ansatz: Solar neutrinos

Alternatively, we are led to the conclusion that only quite specific versions of the ansatz (1.1) for the neutrino masses, where feature *i*) is not valid, can reconcile the indications of atmospheric neutrino oscillations with other neutrino anomalies. We are interested in the specific subset of the mass matrices of type (1.1) that predict the existence of only one neutrino with mass of order m_ν. We relax the hypothesis of *non-singularity* in the “dominant block”, introducing a new small parameter δ, such that

$$\mathcal{M}_{\tau\tau}/\mathcal{M}_{\mu\tau} = \mathcal{M}_{\mu\tau}/\mathcal{M}_{\mu\mu} + \mathcal{O}(\delta)\,;$$

now, an eigenvalue of the dominant block is order $m_\nu \cdot \delta$.

On the theoretical side, we remark that a relatively high degree of tuning is a valuable information on the flavour structure, that goes beyond what is suggested by the NFS or ABE models, and in general by the ansatz considered. Keeping in mind the seesaw [3] formula for the light neutrino masses, $\mathcal{M} = -v^2 Y_\nu^t M_R^{-1} Y_\nu$, such a tuning may suggest us that the determinant of (a block of) the neutrino Yukawa couplings Y_ν is (close to) zero, cfr. [4], or that at leading order only one eigenvalue of the right-handed neutrino mass M_R is sufficiently light to contribute to $\mathcal{M}$, cfr. [19]. The matrix in [4] (second paper), eq. (10), or "texture *II*" in [19], are equivalent to the mass matrix we are considering, in the limiting case $\epsilon = \delta = 0$. Indeed, in [4] the connection between neutrino masses and the flavour problem is also considered. Mechanisms to adequately control the flavour structure of the neutrino mass matrix have also been proposed in [20].

2.1. Formalism

After changing basis in the dominant block, from (ν_τ, ν_μ) to the "heavy" and "light" states (ν_h, ν_l), the neutrino mass matrix becomes:

$$\mathcal{M}' \sim m_\nu \begin{pmatrix} 1 & 0 & \epsilon \\ 0 & \delta & \epsilon \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix}, \tag{2.1}$$

where we assumed that the rotation is not a source of suppression for the $\mathcal{M}'_{el}$ and $\mathcal{M}'_{eh}$ entries (= the *no-special-zeroes* hypothesis). We notice that the heaviest neutrino state, ν_h, can be integrated away, leaving us a two-flavour task; the only effect is that the $\mathcal{M}'_{ee}$ entry receives another contribution, but still of order ϵ^2. Like for eq. (1.4), the mixing of the heaviest neutrino state with the electron neutrino is of order ϵ. The discussion related to eq. (1.5) is still valid.

Oscillations of atmospheric neutrino set the mass scale in eq. (2.1) to $m_\nu^2 \sim \Delta m^2_{atm}$. Now we study the possibility of incorporating also solar neutrino oscillations, taking advantage of the parameter δ. For this sake, we first rewrite the effective mixing matrix between the light states, writing explicitly the coefficients of order unity, p, q and r:

$$\mathcal{M}_{2\times 2} = \sqrt{\Delta m^2_{atm}} \times \begin{pmatrix} p\ \delta, & r\ \epsilon \\ r\ \epsilon, & q\ \epsilon^2 \end{pmatrix}. \tag{2.2}$$

(q is inclusive of the heavier neutrino "seesaw" contribution). The relation between the elements of the neutrino mass matrix and the parameters of oscillation in vacuum is:

$$\begin{cases} \delta m^2\ \sin 2\theta = 2\ \Delta m^2_{atm}\ |r|\ \epsilon\ |p^*\ \delta\ + q\ \epsilon^2|\,, \\ \delta m^2\ \cos 2\theta = \Delta m^2_{atm}\ (|p|^2\ \delta^2 - |q|^2\ \epsilon^4)\,. \end{cases} \tag{2.3}$$

$\delta m^2 \geq 0$ is non-negative, and $\theta \in [0^\mathrm{o}, 90^\mathrm{o}]$.

Two remarks on this result are in order:

i) The physical parameters are 2, but the relevant parameters coming from the ansatz are in general 4: in fact, the modulus of p and q can be incorporated in δ and ϵ, but their relative phase is relevant. However, when the condition $\delta \sim \epsilon^2$ is not fulfilled, this phase is effectively irrelevant.

ii) The propagation in a medium with electrons density ρ_e effectively modifies the second term: $\delta m^2_{\mathit{eff}}\ \cos 2\theta_{\mathit{eff}} = \delta m^2\ \cos 2\theta - 2\sqrt{2}\, E_\nu\, G_F\, \rho_e$. Thus, to have maximal θ_{eff} due to the interplay of vacuum and matter terms [9] we need $\delta \gtrsim \epsilon^2$. Hence, at $\delta = 0$, the MSW mechanism does not work—or in other terms, the case when the singularity of the dominant block is exact (or very pronounced) is a special one.

The allowed values of the oscillation parameters are delimited by the two curves:

$$\delta m^2 = \Delta m^2_{atm}\ \epsilon^2 \times \frac{4\ |r|^2}{\sin^2 2\theta} \times \left[\cos 2\theta \pm 2\epsilon\ \frac{|q|}{|r|} \sin 2\theta\right]. \qquad (2.4)$$

These regions are represented in fig. 1 for two values of ϵ. Allowing for an excursion of a factor of 4 in Δm^2_{atm}, and a factor of 2 in the moduli of the coefficients q and r in previous equation,[2] the region extends by more than two orders of magnitude in δm^2 for any assigned value of θ. The regions close to $\theta = 45^o$ correspond to δ comparable to ϵ^2 (or smaller). The presence of a large mixing is evident from eq. (2.2), since for such a small δ the two neutrino states form a quasi-Dirac neutrino of mass $m_\nu \cdot \epsilon$ (degeneracy being broken at next order in ϵ). In this region, the range of values taken by θ for fixed δm^2 is due to the size of ϵ, from the bracketed term in eq. (2.4); the shrinking from NFS to the ABE value of ϵ results clearly in the figure. The transition from the $\theta \simeq 45^o$ region, to the "typical" small mixing angles, eq. (1.4), takes place for increasing δ's.

Once a solar neutrino solution (identified by δm^2 and $\sin^2 2\theta$) has been selected, one can use eq. (2.4) to find the range of values that are permitted to ϵ. Except when θ is very close to maximal mixing, the second term between brackets in (2.4) can be dropped, and a simple expression for the range $[\epsilon_{min}, \epsilon_{max}]$ can be obtained: this covers many cases of interest.

2.2. Particular solar neutrino solutions

Large angle MSW. The large mixing angle MSW solutions of the solar neutrino problems (both LOW and LMA—we use [21] as a reference analysis and for terminology) are reproduced in the cases illustrated in fig. 1: LOW in both NFS and ABE models, LMA only for the NFS one. Let us recall that the preliminary SK data on the shape of the electron spectrum tend to disfavour the large-angle solutions [22, 21].

[2] We will assume these ranges of values for reference throughout the work.

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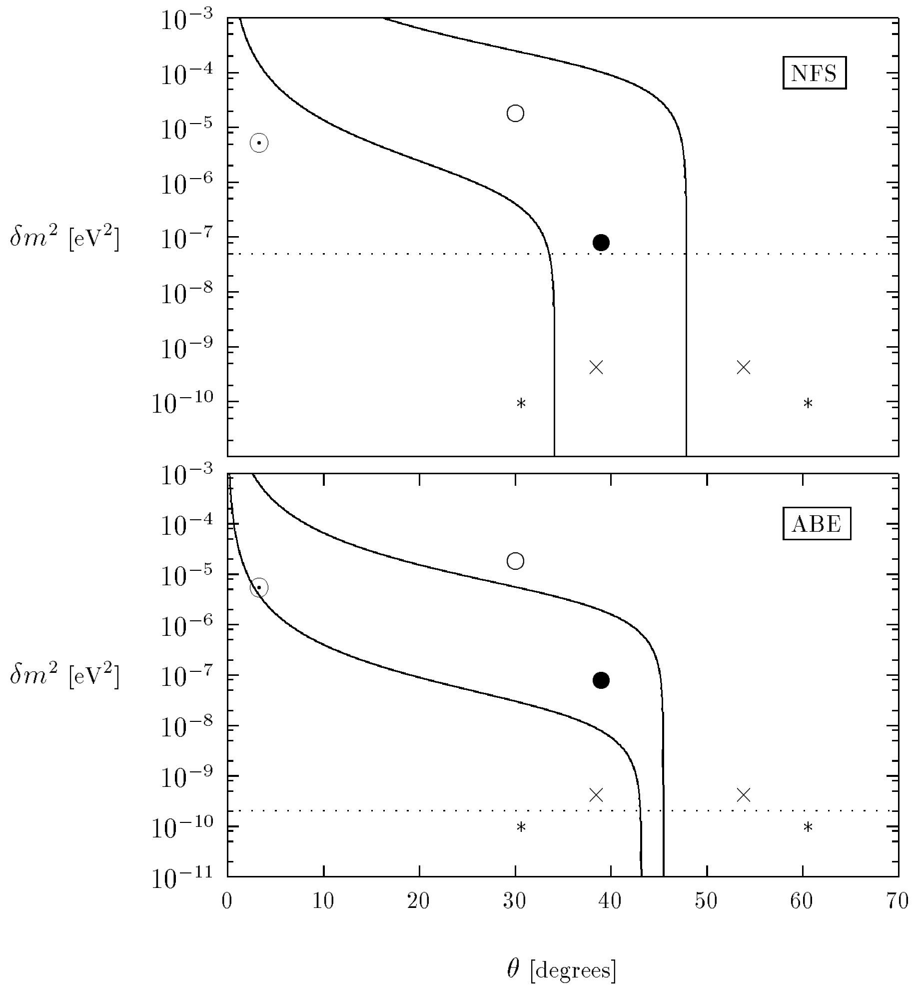

Figure 1: Allowed regions for solar neutrino parameters in the NFS and ABE models. Observed counting rates in solar neutrino experiments can be explained in the vicinity of the indicated points [21]: dotted, void and filled circles correspond to (best fit) MSW solutions denoted as SMA, LMA and LOW. The symbols $\times$ and $*$ indicate two particular VO solutions (see text). The dotted line denotes the lower value of δm^2 compatible with the fine-tuning condition (2.9).

Small angle MSW. It is possible to reproduce the small mixing angle MSW solution (SMA) at best fit values [21] if:

$$\epsilon \sim \sin 2\theta \left(\frac{\Delta m^2_{\odot}}{4\ \Delta m^2_{atm}} \right)^{1/2} = 2 \cdot 10^{-3}. \tag{2.5}$$

The value in (2.5) is close to the value obtained in the ABE model, but much smaller than in the NFS model (see fig. 1). It is very well consistent with:

$$\epsilon = (\sin\theta_C)^4 \text{ or } (m_\mu/m_\tau)^2 \qquad (\text{“small” value})\,. \tag{2.6}$$

Correspondingly to eq. (2.5), we have

$$\delta \sim 5\ \cdot 10^{-2} \simeq (\sin\theta_C)^2.$$

This result, $\delta \sim \sqrt{\epsilon}$, tells us that to obtain the SMA solution the breaking of the hypothesis of *no-singularity* must be weaker than ϵ (more in general, this last condition implies small mixing angles, as it is apparent from eq. (2.2)).

The implementation of the small angle solution is not possible in the NFS model (1.2) without special arrangement in the parameters of order unity: a suppression of r by a power of ϵ ($\sim 1/20$; see eqs. (2.3)) is necessary to hit SMA solution. It is possible in principle that a specific implementation of the NFS model [1] produces a violation of the *no-special-zeroes* hypothesis, but we don't see the reason for that in the present context.

Averaged oscillations. It is interesting to notice that, since maximal mixing ($\sin^2 2\theta \simeq 1$) is permitted, completely averaged neutrino oscillations with survival probability $P(\nu_e \to \nu_e) \simeq 1/2$ [23] are possible for a wide range of values of ϵ, for instance in the ABE and NFS models. Many experimental informations on solar neutrinos can be accounted for in this assumption, with the noticeable exception of the counting rate of the chlorine experiment [24] and of the spectral shape at SK (a detailed study is in [25]).

Vacuum oscillations. Let us consider two vacuum oscillation solutions [26] of particular interest: The best fit global solution, $(\delta m^2, \sin^2 2\theta) = (8 \cdot 10^{-11}\ \mathrm{eV}^2,\ 0.75)$ (global-VO solution, [21]), and the best fit solution to the electron spectral shape only $(\delta m^2, \sin^2 2\theta) = (4.2 \cdot 10^{-10}\ \mathrm{eV}^2,\ 0.93)$ (shape-VO solution, [22], [27] and [28]). To reproduce them exactly, and without large fine-tunings (defined in next paragraph), quite small values of ϵ are required:

$$\epsilon = (\sin\theta_C)^n, \qquad 6 \geq n \geq 4\,; \tag{2.7}$$

global-VO requires $n = 4, 5$ or 6; shape-VO requires $n = 4$ or 5. The corresponding values of δ can be read from eqs. (2.3) (the ϵ^2 term being negligible in these two cases). In the ABE model (n=3) it is possible to reproduce solutions that are allowed at 99% CL, with $\delta m^2 = \mathrm{few} \cdot 10^{-10}$ eV^2 and nearly maximal mixing [21], but not the global-VO or the shape-VO solutions. Many VO solutions can be incorporated in the NFS models (and in the models with even larger value of ϵ) but with very special arrangements of the coefficients of order unity, in the case $\delta \sim \epsilon^2$ considered below.

2.3. Fine-tuning condition

We attempt now an assessment of the likelihood of the various solutions for the solar neutrino problem. To fulfill this program we have to make a priori assumptions, which may be considered unpleasant. But this step is necessary in order to make the model (2.1) predictive: As we saw, treating ϵ and δ as free parameters, *any* solar neutrino solution can be fitted. To fix the ideas we take the NFS (1.2) and ABE (1.3) values for ϵ as reference in the following. A criterion for absence of fine-tuning is that δ should be *at most* of order ϵ, that leads to a lower limit on δm^2:

$$\delta m^2 \gtrsim \Delta m^2_{atm}\ \epsilon^2. \tag{2.8}$$

This condition already gives us a lot of freedom: For the NFS model, the large angle mixing (LMA) is possible; for the ABE model the SMA solution is possible. More specifically, if the origin of the smallness of δ and of ϵ is the same, we may expect $\delta \sim \epsilon$; and using eq. (2.3), we conclude that large angles are to be expected in this assumption. The case $\delta \sim \epsilon$ is interesting, since we can reproduce the LMA solution with a value of ϵ close to the one of the NFS model (1.2). Now, let us consider values $\delta \lesssim \epsilon$. Even in this case, we meet a second (but of course weaker) fine-tuning criterion. As visible in fig. 1, it is possible to obtain δm^2 as small as desired, in the region where the two curves become vertical. This requires, however, a special arrangement between the phases and moduli of p and q, and $\delta \sim \epsilon^2$, as one verifies from eqs. (2.3). The border of the region of parameter space in which it is not necessary to admit this delicate fine-tuning can be estimated assuming $\delta = 0$. Equations (2.3) suggest then:

$$\delta m^2 \gtrsim 2\ \Delta m^2_{atm}\ \epsilon^3. \tag{2.9}$$

Now, also the solution denoted as LOW in [21] can be incorporated by both NFS and ABE models (fig. 1); similarly, it is possible to find agreement with the low δm^2 (and large angle) solutions that are suggested by the analysis of the SK data alone [22]. Averaged solutions are "more natural" for large values of ϵ. The small values of ϵ's that permit to reproduce the VO solutions discussed above (eq. (2.7) are in agreement with the fine-tuning conditions (2.8) *and* (2.9) when $n > 4$. In the case $n = 4$ (that is, eq. (2.6)) the global-VO or shape-VO solutions can be realized only if eq. (2.8) is violated; eq. (2.9) is instead satisfied. The vacuum oscillations solutions with $\theta \approx 45^o$, allowed in the ABE case ($n = 3$) are compatible with eq. (2.9), whereas those in the NFS model should be discarded in this assumption. Of course, these fine-tuning criteria would be irrelevant if a theory of the coefficients of order unity were formulated.[3] *A priori*, one may even conjecture a theory able to explain the delicate cancellation $|p^*\delta + \epsilon^2 q| \ll \epsilon^2$.

Summarizing, the "specific" form of the ansatz–eq. (2.1)–is able to incorporate MSW solutions of the solar neutrino problem, with large (NFS model) or small mixing angle (ABE model), or even other possibilities. We conclude that, in order to state actual predictions, precise values should be given not only for ϵ, but also for the overall mass scale m_ν, for δ and for the coefficients of order unity.

[3] In this respect, a good theoretical question is: How these hypothetical theories relate neutrino masses to other facts, like charged fermion masses, observed matter stability, baryogenesis, *etc.*

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3. Specific forms of the ansatz: LSND anomaly, dark matter

Let us consider now a second possibility, in which we simply neglect the solar neutrino problem. In the case in which the mass scale m_ν is large, we can still take advantage of the presence of large mixing angles in the ansatz (1.4) and explain the atmospheric neutrino problem in terms of oscillations, but now a fine-tuning must be operated on the parameter η:

$$\eta = \frac{\Delta m^2_{atm}}{m^2_\nu}\,.$$

As in section 2, we assume that this arises as a sort of fine structure of the dominant block. In the terminology of section 1.2, we are specifying the general ansatz (1.1) relaxing the *no-degeneracy* hypothesis.

LSND. In the first variant, we can use the small mixing angles (1.4) of the electron neutrino with the heavy states to explain the LSND anomaly [10]. This can be done in the NFS model, if $m_\nu \sim 0.7$ eV ($M_{heavy} \sim 4\cdot 10^{13}$ GeV), that implies for the fine-tuning parameter $\eta = 4\cdot 10^{-3}$. It is not possible, instead, to account for LSND anomaly in the ABE model, since the angle is too small.

Hot dark matter. In the second variant, we consider a larger mass scale, $m_\nu \sim 2.5$ eV, related to two-neutrino hot dark matter component (2νHDM), of an $\Omega = 1$ cosmological model [29]. This assumption is not consistent with the bounds from reactor experiments [30] in the model NFS ($\sin^2 2\theta$ should be reduced by one order of magnitude), whereas the ABE value for ϵ is fine, again because of the small mixing angle. The parameter η and the heavy scale M_{heavy} are smaller than in the previous case.

4. Conclusions and perspectives

The principle of connecting neutrino masses to family structure is very attractive. A related ansatz for massive neutrinos was discussed, both in its generic forms (with parameters m_ν and ϵ) and in its specific form (2.1) (with the additional parameters δ, or η).

The clearest feature of the ansatz considered is about atmospheric neutrinos. Due to the smallness of ϵ, almost pure $\nu_\mu - \nu_\tau$ oscillations take place. Many relevant tests will be possible in the relatively short term:

1. The analysis of low energy induced stopping muons, and partially contained events (presented in preliminary form by the MACRO and SK collaboration [7]).

2. The search of "oscillated" ν_τ flux via neutral current induced events [31]. To perform this inference from observed events, a detailed knowledge of the low

	$CP\!\!\!/$	$\nu_\odot$ SMA	$\nu_\odot$ LMA	$\nu_\odot$ LOW	$\nu_\odot$ VO	$\nu_\odot$ aver.	LSND	2νHDM
ABE	no	yes	no	yes	∼yes	yes	no	yes
NFS	∼yes	no	yes	yes	no?	yes	yes	∼no

Table 1: Alternative facts that can be incorporated by the models NFS (1.2) and ABE (1.3) together with atmospheric neutrino oscillations.

energy neutrino interactions is required.[4] A failure of the detection would rule out the model.

3. The studies of a subdominant electron neutrino component in atmospheric neutrinos, and further reactor studies. Both should give null result, *except if* ϵ *is large, as in eq.* (1.5).

4. A cross-check, or in any case an improvement of our knowledge on the parameter Δm^2_{atm} in long baseline experiments.

5. The search of charged current induced τ's, produced either in artificial beams, or by the atmospheric neutrinos themselves (for some elements for a discussion, see [18]).

A theoretically weak point of the model is that the scale M_{heavy} is fitted, not predicted. However, it is appealing that the scale is $1-2$ orders of magnitude smaller than the (minimal SU(5)) supersymmetric grand unification scale (in the variants considered in section 3, it is even smaller). Since, in a predictive model for the scale, there are typically perturbative couplings y ($y < 1$) such that $v^2/M_{heavy} \equiv y^2v^2/M_X$, the actual scale M_X is expected to be lower than M_{heavy}. Hence, there is *a strong suggestion for the existence of an intermediate mass scale,* possibly right handed neutrinos, even if this is not an unescapable conclusion (for instance, neutrino masses may be related to a "direct" mass term [32]).

We summarize in table 1 other observable features that the ansatz can incorporate, using eqs. (1.2) and (1.3) for ϵ, and using the additional small parameter δ in second to fifth cases, and η in the last two. Other interesting values of ϵ have been discussed in (1.5), (2.6) and (2.7).

The explanation in terms of massive neutrinos of some fact, among *a)* solar neutrino problem (sec. 2), *b)* LSND anomaly (sec. 3) or *c)* dark matter neutrinos (sec. 3), will preclude the explanation of the other ones: the model can be brought to logical contradiction if two (or three) facts were proved to be correct. Among the particular implications, it is interesting that the small mixing angle MSW solution can not be implemented in the NFS model without fine-tunings Indeed, the best implementation of SMA require values of ϵ (and δ) discussed in sec. 2.2, different from those in ABE

[4]New experimental informations will be obtained already at the "close" detector of the K2K experiment [5, 6].

or NFS model. Vacuum oscillation solutions tend to favour quite small values of ϵ (depending sensitively on the values of δm^2 and $\sin^2 2\theta$) or very special arrangements of the coefficients of order unity and $\delta \sim \epsilon^2$ (this corresponds to the question mark in table 1—see the discussion of the fine-tuning conditions (2.8, 2.9)). Models with observable CP violation (in which the other three indications cannot be accounted) are also possible (sec. 1.3).

To increase the predictivity of the ansatz, a detailed theory of the coefficients of order unity, and particularly of the structure of the dominant block is necessary.

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